Mathematics Task Arcs

Overview of Mathematics Task Arcs:

A task arc is a set of related lessons which consists of eight tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Common Core State Standards for Mathematics. In some cases, a small number of related standards from more than one domain may be addressed.

A unique aspect of the task arc is the identification of essential understandings of mathematics. An essential understanding is the underlying mathematical truth in the lesson. The essential understandings are critical later in the lesson guides, because of the solution paths and the discussion questions outlined in the share, discuss, and analyze phase of the lesson are driven by the essential understandings.

The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however the context or representations change.

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Introduction
Addition: Flexible Thinking and Problem-Solving Strategies
A SET OF RELATED LESSONS
Overview

This set of related lessons provides an opportunity for students to develop a deep understanding of addition with two-digit numbers while providing a bridge into addition with three-digit numbers. The related lessons address 2.OA.A.1, 2.NBT.B.6, and 2.NBT.B.7 Content Standards and all of the Mathematical Practice Standards from the CCSSM.

The primary focus of this set of related lessons is the focus on multiple problem-solving strategies. The strategies introduced in this set of lessons include:

- Count On by Tens
- Add by Place Value
- Juggle Amounts to Make Friendly Numbers
- Make a Ten from Ones
- Make Friendly Numbers and Adjust

The tasks are designed so that the identified strategies can be discussed during the Share, Discuss, and Analyze Phase of the lessons. If students have not used the identified strategy during the Explore Phase, the instructor should be prepared to present it during the Share, Discuss, and Analyze Phase of the lesson.

There are six developing understanding tasks and two solidifying understanding tasks in this set of lessons. Each of the developing understanding tasks utilize “put together” situation problems with unknown sums. Though all of the tasks are the same type of situation problem, the complexity of the situations increases over the course of the eight lessons. As students explore the tasks, they are asked to grapple with information in both text formats, as well as simple tables, and finally, more complex tables with multiple pieces of information. Students are expected to “read” tables, select appropriate information from tables, and complete tables by filling in missing information. Students are also expected to make sense of a series of claims using information from a table.

The developing understanding tasks are paired with application opportunities for students to further develop reasoning about the mathematical concept they are exploring. Quick Writes are also provided so that students have an opportunity to construct written responses about the mathematical ideas highlighted in the task.

In Tasks 1 and 2, students explore place value as a means of determining the sums of two-digit numbers and multiples of 10. There are two strategies named in the first two lessons: “Count On by Tens” and “Add by Place Value.” These strategies can be utilized throughout all eight lessons. This is done through the process of solving “put together” situational problems and writing equations that represent the situations. Students are asked to extend their thinking and reasoning about adding two-digit numbers and a multiple of 10 to adding three-digit numbers and multiples of 10.

Then in Task 3, students look for patterns and make sense of the structure of place value by solving a set of addition equations. Task 3, a solidifying understanding task, asks students to analyze sets of equations to determine and write about patterns observed in the sums of the equations as a result of adding multiples of 10 to the same two-digit addend. In this task, students work with two-digit and three-digit addends.
Tasks 4, 5, 6, and 7 are designed for students to continue working with place value as a means for solving addition problems, while also exploring the concepts of conservation and compensation. In these lessons, students investigate conservation while considering how moving amounts from one addend to another addend makes addition easier. The strategies associated with conservation and utilized in these lessons are: “Juggle Amounts to Make Friendly Numbers” (introduced in Task 4) and “Make a Ten out of Ones” (introduced in Task 5). The strategy associated with compensation and utilized in the lessons is “Make Friendly Numbers and Adjust” (introduced in Task 6). The corresponding tasks, Tasks 4, 5, 6, and 7, are all developing understanding tasks framed as “put together” situational problems. Task 4 and Task 5 present information in the form of a table. Task 6 also presents information in the form of a table, but students must be more attentive to the structure of the table and select the appropriate information to reason about a claim. Task 7 presents information in the form of a table as well, but in this case, students are expected to complete the table by filling in the missing information.

Task 8 is designed to pull together all of the threads running through the prior seven lessons. Task 8 is a solidifying task that asks students to analyze a series of equations, determine how the original equation changes through the series, and identify the strategy that was applied that resulted in the changes to the equation. Then students solve equations by selecting and applying the strategies developed throughout the set of lessons.

The prerequisite knowledge necessary to enter these lessons is a basic understanding of addition, knowledge about place value (ones, tens, and hundreds), and the format of addition equations. Students would benefit from being familiar with part-part-whole relationships, part-part-whole maps, base ten blocks, and hundreds charts. (Blackline masters of a part-part-whole map, base ten blocks, and a hundreds chart are provided following the progression chart.)

Through engaging in the tasks in this set of related lessons, students will:

- solve “put together” situational problems using base ten blocks and/or a hundreds chart;
- determine if the “unknown” in a situational story task represents a part or a whole;
- understand that tables can be “read” and can represent part-part-whole relationships;
- apply strategies (listed previously) to solve addition problems; and
- use a variety of representations to respond to situational story problems.

By the end of these tasks, students will be able to answer the following overarching questions:

- What strategies can we use to solve addition situations?
- What are different problem-solving strategies for solving two-digit and three-digit addition problems?
- How are the strategies similar to and different from each other?
- What happens when we keep adding 10 to a number?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning’s expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.
<table>
<thead>
<tr>
<th>Represent and solve problems involving addition and subtraction.</th>
<th>Essential Understandings</th>
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</thead>
<tbody>
<tr>
<td>2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *Greyed-out portions not addressed in task or lesson.</td>
<td>Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>CCSS for Mathematical Content: Operations and Algebraic Thinking AND Number and Operations in Base Ten</th>
<th>Essential Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Use place value understanding and properties of operations to add and subtract.</strong></td>
<td>Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.</td>
</tr>
<tr>
<td>2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
<td>The position of the digits 0–9 in a number represents different size amounts within a written number.</td>
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<tr>
<td>2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.</td>
<td>Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.</td>
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<td>Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.</td>
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<td>A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.</td>
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<td>Two or more expressions/equations are equivalent if they represent the same quantity.</td>
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<td>Altering a number in an addition equation by increasing or decreasing an addend means that the sum will also have to be altered in order to compensate by using the operation opposite from the initial increase or the decrease in order to maintain the original amounts. (e.g., When adding 69 + 22, 69 can be increased to 70, then 70 + 22 = 92 but increasing one addend up by 1 means that 1 must be subtracted from the sum 92 to compensate for the 1 added to 69.)</td>
</tr>
</tbody>
</table>
The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Tasks’ CCSSM Alignment

<table>
<thead>
<tr>
<th>Task</th>
<th>2.OA.A.1</th>
<th>2.NBT.B.6</th>
<th>2.NBT.B.7</th>
<th>MP 1</th>
<th>MP 2</th>
<th>MP 3</th>
<th>MP 4</th>
<th>MP 5</th>
<th>MP 6</th>
<th>MP 7</th>
<th>MP 8</th>
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<td>Task 4 Sit-Ups</td>
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<td>Task 5 Flowers</td>
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<td>Task 7 Fieldtrip to the Zoo</td>
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## Lesson Progression Chart

### Overarching Questions
- What strategies can we use to solve addition situations?
- What are different problem-solving strategies for solving two-digit and three-digit addition problems?
- How are the strategies similar and different from each other?
- What happens when we keep adding 10 to a number?

<table>
<thead>
<tr>
<th>Content</th>
<th>Task 1 Prizes for the Class Developing Understanding</th>
<th>Task 2 Collections Developing Understanding</th>
<th>Task 3 What’s Changing? Solidifying Understanding</th>
<th>Task 4 Sit-Ups Developing Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Explores place value, adding 10 and groups of 10 and adding numbers according to their place value by solving “put together” situational problems, whole unknown.</td>
<td>Establishes understanding of place value, adding 10 and groups of 10 and adding numbers according to their place value by solving “put together” situational problems, whole unknown.</td>
<td>Solidifies understanding of place value, adding 10 and groups of 10 by solving addition equations, sum unknown. Recognizes that only the tens place changes when adding 10.</td>
<td>Solves “put together” situational problems, whole unknown. Explores conservation, moving amounts between addends without changing the sum by solving “put together” situational problems, whole unknown.</td>
</tr>
<tr>
<td>Strategy</td>
<td>Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.</td>
<td>Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.</td>
<td>Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.</td>
<td>Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers (moves amounts from one addend to another and adds).</td>
</tr>
<tr>
<td>Representations</td>
<td>Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
<td>Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
<td>Starts with a series of equations and asks students to solve for the sum. Asks students to analyze sets of equations and to determine how the sums change. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent addition equations.</td>
<td>Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
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<td>Task 5</td>
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<td><strong>Flowers</strong></td>
<td><strong>Sports Equipment</strong></td>
<td><strong>Fieldtrip to the Zoo</strong></td>
<td><strong>Flexible Thinking</strong></td>
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<td>Developing Understanding</td>
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<td>Add by Place Value to combine like amounts.</td>
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<td>Add by Place Value to combine like amounts.</td>
<td>Add by Place Value to combine like amounts.</td>
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<tr>
<td>Make a Ten from Ones (uses the ones to make a ten and then adds).</td>
<td>Make a Ten from Ones.</td>
<td>Make a Ten from Ones.</td>
<td>Make a Ten from Ones.</td>
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<tr>
<td>Make Friendly Numbers and Adjust (increases an addend, adds and then adjusts the resulting sum).</td>
<td>Make Friendly Numbers and Adjust.</td>
<td>Make Friendly Numbers and Adjust.</td>
<td>Make Friendly Numbers and Adjust.</td>
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<tr>
<td><strong>Representations</strong></td>
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<tr>
<td>Starts with context presented in the form of a table.</td>
<td>Starts with context presented in the form of a table.</td>
<td>Starts with context presented in the form of a table.</td>
<td>Asks students to analyze sets of expressions, determine what changed in the sets, and then apply the same reasoning to another set of expressions.</td>
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<tr>
<td>Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
<td>Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
<td>Asks students to complete the table by filling in the missing sums.</td>
<td>Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
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<td>Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.</td>
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Part-Part-Whole Map
# Hundreds Chart

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</tr>
</tbody>
</table>
### Base Ten Blocks

#### Ones

![Ones](image1.png)

#### Tens

![Tens](image2.png)
Tasks and Lesson Guides
Addition: Flexible Thinking and Problem-Solving Strategies
A SET OF RELATED LESSONS
Prizes for the Class

Mattie and Kyle are second grade students. There are 50 students in second grade. Mattie and Kyle both bring prizes for the students.
- Mattie brings 27 whistles and 20 stickers.
- Kyle brings 27 whistles and 30 stickers.

Which student brings enough prizes for the second grade students? Explain how you know using models and equations. Be prepared to share your thinking with the class.
## Prizes for the Class

**Rationale for Lesson:** Students solve “putting together” situational story problems with an unknown whole (sum), while exploring adding groups of tens by adding on 10 fingers, 10 counters, 10 squares or 1 row on a hundreds chart, a ten rod, or decomposing a two-digit number by place value and recombining by adding the tens and then the ones. Two strategies are named in this lesson, “Count On by Tens” and “Add by Place Value,” and should be added to the strategies chart during the Share, Discuss, and Analyze Phase of the lesson.

**Task 1: Prizes for the Class**
Mattie and Kyle are second grade students. There are 50 students in second grade. Mattie and Kyle both bring prizes for the students.
- Mattie brings 27 whistles and 20 stickers.
- Kyle brings 27 whistles and 30 stickers.

Which student brings enough prizes for the second grade students? Explain how you know using models and equations. Be prepared to share your thinking with the class.

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>2.OA.A.1</th>
<th>2.NBT.B.6</th>
<th>2.NBT.B.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
<td></td>
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</tbody>
</table>

**Standards for Mathematical Practice**
- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
### Essential Understandings

*Greyed-out portions not addressed in task or lesson.*

- Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.
- Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.
- Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.

### Materials Needed

- Student reproducible task sheet
- Part-part-whole map, hundreds chart, base ten blocks
SET-UP PHASE
Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes. Use the counters and your part-part-whole mapping device or create a diagram to show the prizes. Write equations to tell about the prizes Mattie and Kyle bring to class.

EXPLORE PHASE

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t get started.</td>
<td>What do you know about Mattie’s prizes?</td>
<td>How can you show Mattie’s prizes with counters?</td>
</tr>
<tr>
<td>Uses mapping device and base ten blocks.</td>
<td>How does your model show the prizes?</td>
<td>Write an equation to describe Mattie’s prizes. When you finish, write an equation to describe Kyle’s prizes.</td>
</tr>
<tr>
<td>Mattie</td>
<td></td>
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</tr>
<tr>
<td>Kyle</td>
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<tr>
<td>Has a correct sum for each students’ collection.</td>
<td>Tell me how you arrived at the total number of prizes.</td>
<td>Can you write an equation that describes Mattie’s total number of prizes?</td>
</tr>
<tr>
<td>Writes equations but has not answered the question.</td>
<td>Who has enough prizes for the students in second grade? How do you know?</td>
<td>Write your explanation on your paper so that anyone reading your paper knows who has the most prizes and how you know this.</td>
</tr>
<tr>
<td>1. Mattie: 27 + 20 = 47</td>
<td></td>
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<tr>
<td>2. Kyle: 27 + 30 = 57</td>
<td></td>
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</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Answer the main question:

- Which student has enough prizes for the second grade students? (Mattie didn’t have enough; she only had 47 prizes. Kyle had enough for everyone; he had 57 prizes.)

Mattie’s prizes

EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

- Tell us about Mattie’s prizes. Tell us and show us how you found the total. (I showed 27 and 20.)
- Say more about the 27 and 20. Are you telling us the total or parts of the total amount of prizes? (Some are whistles and some are stickers.)
- I noticed that you referred specifically to the prizes. This really helps us be clear about which prizes we are talking about. (Marking)
- So you are saying that we know two parts. One of the parts is 27, this part is the number of whistles, and another part is 20, this part is the number of stickers. (Revoicing)
- What are we trying to find? (How many altogether.)
- Okay, so we know the parts and need to find the sum of the parts.

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.

Count On by Tens, using base ten blocks:

- How many prizes did Mattie bring? How did you get that answer? (I added 27 and 10 more and then 10 more, and got 47.)
- You said you added 10 and then added 10 more on to the 27. Where did you see 10 in the story problem? (There are two tens in the 20.)
- I don’t think I understand. Can someone say more about the tens inside the 20? (There is a 2 in the tens place, that means there are two tens.)
- You are right. This 2, in 20, means two tens. So you thought of 20 as 10 and 10. Then you started with the 27 and counted on two tens. You said 27, 37, 47. (Revoicing and Marking)

Count On by Tens, using the hundreds chart:

- Did anyone think about 20 this way but make use of the hundreds chart? (I did. I started at 27 and moved down one row to 37 and then down another row to 47.)
- Can someone explain why he “moved down one row from 27?” What does that mean? (Each row is 10, you can count each square and get 10 or just move down one row and one row is 10.)
- When we use what we know about place value, we know that 20 is the same as 10 and 10. Then we were able to count on by tens from 27. We’ll add this strategy to our strategies chart (add Counting On by Tens). This can make adding on numbers that are groups of 10 easier. (Recapping)

Count On by Tens, applying strategy to numbers in the hundreds:

- Let’s try adding on tens to a number in the hundreds. Let’s do 127 + 20 = ______ off the side here. (Challenging) (127, 137, 147)
- We did 27, 37, 47; then we did 127, 137, 147. Did it work the same way? Turn and talk with a friend.
- Over the next couple of days, we are going to keep talking about tens and adding on groups of 10 to a number to make adding easier. (Marking)
EU: Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.

Add by Place Value, using base ten blocks:

- Did anyone find Mattie’s total number of prizes a different way? (I add all the tens and then the ones.)
- Say more about adding all of the tens in both of the sets of prizes. (There are two tens in 27 and two tens in 20, so I added the tens. There are four. Then I added on the seven ones.)
- There are four what? Can you show us with the rods and ones? (Not four, like 1, 2, 3, 4, there are four tens. There are two tens here and two tens here, so there are four tens.)
- You were very precise when you explained what you saw. It helped when you said two tens and two tens is four tens. (Marking)
- Who can say back what he said about the tens and explain how he added them? (10, 20, and then 30 and 40.)
- So you saw two tens and two tens, which is four tens, 40 prizes altogether. Because you are adding tens you can skip count by 10. Then you added the seven ones and found the sum 47. This is a really important strategy to remember, so we should write it down on our strategies chart. We are going to be using it again. Let’s call this strategy, “Add by Place Value,” since you added the tens together first and then the ones. (Revoicing and Marking)

Add by Place Value, applying strategy to an equation:

- Let’s see if we can see the tens and ones when we look at the expression. Where are the tens and ones in 27 + 20? (The 2 in 27 is two tens and then two more tens is four tens.)
- Yes, there are two tens here, and two tens here, that is 40. So the 4 means four tens or 40. Then we need the seven ones. (Write 40 + 7 = 47.) The sum is 47. (Revoicing and Marking)
- How did using the base ten blocks help us understand what she was doing with the tens and ones? (We could really see the two tens and the two tens making four tens. It was easier to think about the four and seven because there are four tens and seven ones.)
- We used place value when we added the tens in both numbers together and then added the ones to find the sum. The numbers in the tens place can be added together because they represent the groups of 10. The numbers in the ones place can be added together because they represent the ones. Then the tens and ones can be added together. (Revoicing and Marking)

Repeat the discussion cycle and sequence of EUs for Kyle’s prizes.

Application

Solve the problems. Make use of either the “Count On by Tens” or the “Add by Place Value” strategies.

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<tbody>
<tr>
<td>58 + 10 =</td>
<td>43 + 30 =</td>
<td>164 + 10 =</td>
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<td>58 + 20 =</td>
<td>43 + 40 =</td>
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<tr>
<td>58 + 30 =</td>
<td>43 + 50 =</td>
<td>164 + 30 =</td>
</tr>
</tbody>
</table>
| **Summary** | When we solve story problems, we need to know if we are looking for the whole or a part.  
When we add groups of tens, we can count on by tens.  
When we add two-digit numbers, we can use place value to find the sums by adding all of the tens together and all of the ones together, and then add those amounts to find the sum. |
| **Quick Write** | Look at the sums for the applications problems and write about what you notice. |

**Support for students who are English learners (EL):**
1. Slow down discussions for students who are English learners by asking other students to repeat ideas, to put ideas in their own words, and continually pointing to the model and the equation simultaneously.
2. Publicly list the problem-solving strategies of “Count on by Tens” and “Add by Place Value.”
Name_________________________________________________________

**Collections**

Felix collects pencils and erasers.
- He has 54 blue pencils and 30 yellow pencils.
- He has 34 sports erasers and 50 animal erasers.

Felix claims that he has more pencils than erasers. Do you agree with Felix’s claim that he has more pencils than erasers?

Explain your thinking by making models or creating a diagram. Write equations to show Felix’s pencils and erasers. Be prepared to share your thinking with the class.
Collections

Rationale for Lesson: In this lesson, students are asked to determine which of two combinations of sets have the most and to justify how they know the set has the most. Students solve “putting together” situational problems with an unknown whole (sum) by adding groups of tens on to a two-digit number or by decomposing a two-digit number by place value and recombining the tens and then the ones, and then adding the tens and the ones. The two strategies discussed in this lesson are: “Count On by Tens” and “Add by Place Value.” Students also discuss equivalency and consider what makes two expressions equivalent.

Task 2: Collections
Felix collects pencils and erasers.
• He has 54 blue pencils and 30 yellow pencils.
• He has 34 sports erasers and 50 animal erasers.
Felix claims that he has more pencils than erasers. Do you agree with Felix’s claim that he has more pencils than erasers?
Explain your thinking by making models or creating a diagram. Write equations to show Felix’s pencils and erasers. Be prepared to share your thinking with the class.

Common Core Content Standards
*Greyed-out portions not addressed in task or lesson.

| 2.OA.A.1 | Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| 2.NBT.B.6 | Add up to four two-digit numbers using strategies based on place value and properties of operations. |
| 2.NBT.B.7 | Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. |

Standards for Mathematical Practice

| MP1 | Make sense of problems and persevere in solving them. |
| MP2 | Reason abstractly and quantitatively. |
| MP3 | Construct viable arguments and critique the reasoning of others. |
| MP4 | Model with mathematics. |
| MP6 | Attend to precision. |
| MP7 | Look for and make use of structure. |
| MP8 | Look for and express regularity in repeated reasoning. |
### Essential Understandings

*Greyed-out portions not addressed in task or lesson.

- Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.
- Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.
- Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.
- Two or more expressions/equations are equivalent if they represent the same quantity.

### Materials Needed

- Student reproducible task sheet
- Part-part-whole map, hundreds chart, base ten blocks
**SET-UP PHASE**

Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes. Use the counters and your part-part-whole mapping device or create a diagram to show the pencils and erasers. Write equations to tell about Felix's pencils and erasers.

**EXPLORE PHASE**

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<thead>
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<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Can’t get started.</td>
<td>What do you know about Felix’s pencils and erasers?</td>
<td>How can you show Felix’s collection of pencils and erasers with the base ten blocks?</td>
</tr>
<tr>
<td>Creates a diagram or builds a model.</td>
<td>How do the blocks tell about Felix’s pencils? Erasers?</td>
<td>Write an equation that tells about Felix’s total number of pencils. Write an equation that tells about Felix’s total number of erasers.</td>
</tr>
<tr>
<td>Uses a hundreds chart.</td>
<td>Show us how you used the hundreds chart to think about the total number of pencils or erasers.</td>
<td>Write an equation to describe what you did on the hundreds chart to determine the total number of pencils. Do the same for the erasers.</td>
</tr>
<tr>
<td>Writes equations, but does not answer the question.</td>
<td>Did you answer the question in the task? If so, how?</td>
<td>Write an explanation so that anyone reading your paper understands the answer to the question.</td>
</tr>
</tbody>
</table>

1. Pencils: 54 + 30 = 84
2. Erasers: 34 + 54 = 84
SHARE, DISCUSS, AND ANALYZE PHASE

Consider Felix’s claim:

- Now that we’ve all had a chance to think about Felix’s claim, what do you think? Do you agree that he has more pencils than erasers? *(He is wrong; he has the same amount of both. I disagree with Felix; he does not have more pencils. I think Felix is right, he has more blue pencils than anything else.)*
- When we agree or disagree with a claim that someone makes, we have to support our thinking. So let’s talk about why we disagree with Felix.

Felix’s total number of pencils

EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

- Tell us about Felix’s pencils. *(He has 54 blue pencils and 30 yellow pencils.)*
- What are we trying to find out about Felix’s pencils? *(His total number of pencils.)*
- We know he has 54 blue pencils and 30 yellow pencils and need to find the total number of pencils. That means we know the parts, the addends, and we need to find the whole, the sum. *(Revoicing)*

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.

*Count On by Tens, using the place value of tens and ones:*

- How many pencils does Felix have? *(84)* How did you find the sum of 84? *(I thought of 30 as 10, 10, and 10, so I counted on three tens.)*
- I’m confused. Can you say more? I know you have three tens, but you didn’t tell me from what number you counted on by three tens from. *(I started from the 54 and said 64, 74, 84.)*
- You saw 3 in the tens place of 30 and knew you could count from 54, three more tens.
- Did anyone else think this way? Can everyone count on three more tens from 54? *(54: 64, 74, 84)*

*Count On by Tens, using the hundreds chart:*

- How can you figure this out by using the hundreds chart? *(I counted down three rows on the hundreds chart. Each time I moved down a row, I landed on the same number she counted.)*
- I don’t think I understand. Who understands and can come up to the hundreds chart and show us what you would do? *(She started at 54 on the hundreds chart. Then she went down to 64, then down again to 74, and then down again to 84.)*
- Who understands and agrees with her? How do you know 84 is the sum of 54 + 30? *(54 is where we started and then 10 more was 64, then 10 more 74, and 10 more 84. This is 30 more.)*
- Why doesn’t the 4 ever change in 54 when 30 is added to 54? Why does the 5 in 54 change? *(The 5 in 54 changes because only tens are added and 5 is in the tens place.)*
- Thank you for being so precise. You said where you started, you started at 54 on the hundreds chart. Then you moved down 30 squares, to 84. *(Marking)*
### Comparison of Two Problem-Solving Strategies Above

**EU: Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.**

*Add by Place Value, relating strategies: Counting On by Tens to Add by Place Value:*

- Who can say how our use of the hundreds chart and the base ten blocks was the same? *(Challenging)* *(We counted on by tens on the hundreds chart and with the rods.)*
- You are right; we used different tools but our strategy was the same. We were using our Counting On by Tens strategy. We both started with 54 and added 3 tens to make 84 because the 3 in 30 is 3 tens. *(Marking and Recapping)*
- Who can tell us how counting on by tens is similar to using the “Add by Place Value” strategy? *(We are adding the 3 tens in 30 to the 5 tens in 54 to get 84.)* *(Challenging)*

**EU: Two or more expressions/equations are equivalent if they have represent the same quantity.**

- We did 54 + 30 and found we have 84 pencils. Can anyone look at 34 + 50 and immediately tell me the sum? *(84 erasers.)*
- I noticed that you said erasers. This was helpful because I won’t get confused and think that I am talking about pencils. *(Marking)*
- How do you know that there are 84 erasers? *(The four just changed from the 54 to the 30. Other than this, there are 50 + 30 erasers and 50 + 30 pencils.)*
- Who understands what she said and can say it in their own words?
- Can we write 54 + 30 = 34 + 50? If so, why? *(Challenging)* *(Both are 84.)*
- We would get the same sum, 84. How did you figure that out? *(There are 34 erasers and 50 erasers, so if we take just the tens, there are five tens and three tens just like the pencils, that makes eight tens, then we have four ones, so there are 84.)*
- Look at the colored chalk that I used to show the 50 on each side of the equal sign and the 30 on each side of the equal sign. So the expressions are equivalent because the same amount is on each side of the equal sign. Each side has 84; this side shows 84 erasers and this side shows 84 pencils. *(Marking)*

### Application

Solve each pair of equations below. Use base ten blocks, your hundreds chart, or what you know about place value to solve the equations. Think about the strategies we have added to our chart.

| 48 + 20 = ____ | 63 + 30 = ____ | 56 + 40 = ____ | 145 + 30 = ____ |
| 28 + 40 = ____ | 33 + 60 = ____ | 46 + 50 = ____ | 135 + 40 = ____ |

What do you notice about each pair of equations?

### Summary

When we solve story problems, we need to know if we are looking for the whole or a part.

When we add groups of tens, we can count on from the starting amount by tens.

When we add two-digit numbers, we can use place value to find the sums by adding all of the tens together and all of the ones together, and then add those amounts to find the sum.
<table>
<thead>
<tr>
<th><strong>Quick Write</strong></th>
<th>Write an explanation to tell why these two equations have the same sum.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>71 + 20 = _____</td>
</tr>
</tbody>
</table>

**Support for students who are English learners (EL):**

1. Slow down discussions for students who are English learners by asking other students to repeat ideas, to put ideas in their own words, and continually pointing to the model and the equation simultaneously.

2. Add to the running list of solution paths for adding two-digit numbers and ask students who are English learners to step back and notice similarities, differences, and/or patterns. In this lesson, problem-solving strategies referenced are “Count on by Tens” and “Add by Place Value.”
What’s Changing?

1. Solve the sets of addition equations with two-digit numbers below. Use or draw base ten blocks or use a hundreds chart to solve the problems.

<table>
<thead>
<tr>
<th>Set A</th>
<th>Set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>43 + 20 = _____</td>
<td>52 + 10 = _____</td>
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<tr>
<td>43 + 30 = _____</td>
<td>52 + 20 = _____</td>
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<tr>
<td>43 + 40 = _____</td>
<td>52 + 30 = _____</td>
</tr>
<tr>
<td>43 + 50 = _____</td>
<td>52 + 40 = _____</td>
</tr>
</tbody>
</table>

2. Study the sums in each set. What is similar about the sums in each set? What is different? What is happening with the sums?
3. The equations in Set C have been solved for you. What is changing in all of the sums?

<table>
<thead>
<tr>
<th>Set C</th>
</tr>
</thead>
<tbody>
<tr>
<td>145 + 10 = 155</td>
</tr>
<tr>
<td>145 + 20 = 165</td>
</tr>
<tr>
<td>145 + 30 = 175</td>
</tr>
<tr>
<td>145 + 40 = 185</td>
</tr>
</tbody>
</table>

4. Apply the same reasoning and solve these sets of equations below.

<table>
<thead>
<tr>
<th>Set D</th>
<th>Set E</th>
</tr>
</thead>
<tbody>
<tr>
<td>123 + 10 = ____</td>
<td>135 + 20 = ____</td>
</tr>
<tr>
<td>123 + 20 = ____</td>
<td>135 + 30 = ____</td>
</tr>
<tr>
<td>123 + 30 = ____</td>
<td>135 + 40 = ____</td>
</tr>
<tr>
<td>123 + 40 = ____</td>
<td>135 + 50 = ____</td>
</tr>
</tbody>
</table>
What’s Changing?

**Rationale for Lesson:** In this lesson, students solidify their understanding of adding on groups of 10 to a two-digit number and discover that the addition problems can be solved by counting on from the initial amount by tens or by adding the amounts in the tens place and tacking on the ones. Students also discover that this pattern related to the tens changing only continues when there are less than nine tens and less than nine ones. The two strategies discussed in this lesson are: “Count On by Tens” and “Add by Place Value.” Students also discuss equivalency and consider what makes two expressions equivalent.

**Task 3: What’s Changing?**

1. Solve the sets of addition equations with two-digit numbers below. Use or draw base ten blocks or use a hundreds chart to solve the problems.

   - **Set A**
     - 43 + 20 = ____
     - 43 + 30 = ____
     - 43 + 40 = ____
     - 43 + 50 = ____

   - **Set B**
     - 52 + 10 = ____
     - 52 + 20 = ____
     - 52 + 30 = ____
     - 52 + 40 = ____

2. Study the sums in each set. What is similar about the sums in each set? What is different? What is happening with the sums?

See student paper for complete task.

**Common Core Content Standards**

*Greyed-out portions not addressed in task or lesson.

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>2.NBT.B.6</th>
<th>2.NBT.B.7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
</tr>
</tbody>
</table>

**Standards for Mathematical Practice**

| MP1 | Make sense of problems and persevere in solving them. |
| MP3 | Construct viable arguments and critique the reasoning of others. |
| MP4 | Model with mathematics. |
| MP6 | Attend to precision. |
| MP7 | Look for and make use of structure. |
| MP8 | Look for and express regularity in repeated reasoning. |
| Essential Understandings | • Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.  
• The position of the digits 0–9 in a number represents different size amounts within a written number.  
• Two or more expressions/equations are equivalent if they represent the same quantity.  
• Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.  
• Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same. |
| --- | --- |
| Materials Needed | • Student reproducible task sheet  
• Part-part-whole map, hundreds chart, base ten blocks |
**SET-UP PHASE**
This task is a little different from others that we have done. Put your finger on the first set of problems, Set A. You have to solve each of these problems by using base ten blocks, drawing a picture, or using the hundreds chart. Then you do the same thing for Set B. When you are done solving all of the problems, answer the question at the bottom. Then move on to study Set C, and see if you can apply the same reasoning to Set D and Set E. Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes and then work with a partner.

**EXPLORE PHASE**

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t get started.</td>
<td>How could you show us 43 + 20 with the rods and ones?</td>
<td>How can this help you figure out the rest of the problems in Set A?</td>
</tr>
<tr>
<td>Inaccurately represents 43 + 30.</td>
<td>Tell me what you have shown with the rods and ones for 43 + 30. What does this 3 in 30 mean? What does this 3 in 43 mean? Why do they mean two different things?</td>
<td>Now that you know that the 3 in 43 means three ones, and that the 3 in 30 means three tens, show me 43 + 30 with the base ten blocks, with the ten rods and ones.</td>
</tr>
<tr>
<td>Applies the “Count On by Tens” strategy to find the sum for each addition equation.</td>
<td>Look at the sums you have written. 43 + 10 = 53 43 + 20 = 63 43 + 30 = 73 You counted by tens to find the sum for each problem. What do you notice is changing with the sums 53, 63, and 73? Why is this happening?</td>
<td>Move on to the next set of equations. See if when you add on groups of ten, if the same thing you noticed about the sums in Set A happens in Set B. If it does happen, write a sentence about your observation.</td>
</tr>
<tr>
<td>Possible Student Pathways</td>
<td>Assessing Questions</td>
<td>Advancing Questions</td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Applies the “Count On by Tens” strategy using the hundreds chart.</td>
<td>Show us how you used the hundreds chart to solve the equations.</td>
<td>What is the sum of the next equation?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>How can you use what you know about the first equation to solve the second equation?</td>
</tr>
<tr>
<td>Attempts to apply the “Add by Place Value” strategy but adds the number in the tens place to the number in the hundreds place in Part D (error).</td>
<td>How did you find the sums of the problems in Set D?</td>
<td>Write an explanation that tells anyone who reads your paper how you found the sums.</td>
</tr>
<tr>
<td></td>
<td>How could you show what you did using the hundreds chart, tens rods, and ones to figure out the solution?</td>
<td></td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Solutions for Problems in Set A

EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

Count On by Tens, using the hundreds chart:
- Let’s take a look at the first equation from Set A. What is the sum of the first problem? (63)
  How did you find 63? (I started at 43 and added 20 more to find 63.)
- Can someone show us what this would look like using the hundreds chart? (I started at 43 and dropped to 63.)
- So one of the parts you are adding is the 43 and the other part is 20; that’s why you said 20 more, right? The sum of 43 + 20 is 63. (Marking)

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.

Count On by Tens, using the hundreds chart:
- How did you know to drop to 63 on the hundreds chart? (We need to add 20 to 43, so I add 10 and 10 more, which is two rows.)
- If we start at 43 and add on 10 we have 53, then we add on 10 more and have 63. (Revoicing)
- Show us 43 + 30.
- How many rows did you drop down and how do you know? (I dropped down three rows because three tens is 30.)
- What is 43 + 40? (83) What is 43 + 50? (93) What is 43 + 60? (103)

EU: The position of the digits 0–9 in a number represents different size amounts within a written number.
- What patterns do you notice when you look at 43 + 30, 43 + 40, and 43 + 50? (Challenging)
  (We go up by 10 each time a 10 is added.)
- When 10 more is added to one of the addends, the sum increases by 10. (Revoicing)
- Who understands what he is saying about the set of problems and can say more?
- What happened when we got to 43 + 60? (It turned into 103.)
- Why did this happen? (It reached nine tens and now we have 10 tens.)
- How much is 10 tens? Turn and talk with your partner. (10 tens is 100 and then three more is 103.)
- If 43 + 60 is 103, what is 43 + 70? Stop and jot.
- Great work, 43 + 70 is 11 tens and three ones, or 113. (Marking)
EU: Two or more expressions/equations are equivalent if they represent the same quantity.

- Let’s continue to talk about 73 + 40. Would I be permitted to write 73 + 40 instead of 43 + 70? Turn and talk with a partner.
- Are these two expressions equivalent, 73 + 40 = 43 + 70? (Yes, both sides of the equal sign have 70 + 40 + 3 and both sides equal 113.)
- Was 73 + 40 easier than 43 + 70 for anyone? What made this easier for you? (I checked the answer by doing 73, 83, 93, 103, and 113.)
- So you counted on four tens as a way of checking on the sum. Why didn’t you count on seven tens from 43 to check 43 + 70? (Seven tens is a lot to count on from but just counting on by four tens isn’t as much.)
- So we have the right to switch the tens if we want. We changed 43 + 70 to 73 + 40 so that the 70 would be first. We also learned that sometimes a strategy like counting on from an addend is easier to use when counting on from the largest addend. 73 is the largest addend we can make from the amounts we have, and then we only had to count on four tens.

(Recapping)

Discuss the solutions and patterns in Set B, then discuss the Pattern of Adding Tens to an Addend

EU: Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.

Comparing the equations in Set A and Set B: the pattern of adding tens to an addend:

- What is the same between the problems in Set A and B? (Tens are being added in each set of problems.)
- When a 10 is added to the two-digit number, what do you notice happens to the sum? (It goes up by 10, too.)
- Someone else put this idea about adding tens and the relationship to the sum into your own words. (When you add 10 to a number, then the sum goes up by 10.)
- Look at these equations. What is the sum of the last expression?

52 + 40 = 92
52 + 50 = 102
52 + 60 = ___

- Right now we are saying that we see the tens change when 10 is added to a two-digit number, we are saying that the sum increases by 10, also. (Marking)

Analyzing the equations in Set C, connecting to “Add by Place Value” strategy:

- How are the equations in Set C different from the equations in Set A and Set B? (There is a number with hundreds.)
- What is happening with the sums in Set D? (They are all going up by one.)
- Say more about how they are going up by ones. I want to make sure we all understand what you mean. (155, 165, 175, 185, the middle number goes up by one.)
- When you said goes up by one, you mean that the sum goes up by one 10. How do we know that it is one 10 and not just one? (Revoicing) (The number that goes up is in the tens place, so it is one 10.)
- Why is the sum going up by 10 each time? (Each time another 10 is added. The first time 20 is added, then the next time 30 is added, 30 is 10 more than 20.)
- Did we see this happening in any of the other sets? Where?
- But this set has three-digit numbers, and we are adding in the hundreds. How can the same thing be happening with the numbers in the tens place? (Challenging) (We are only adding tens, we aren’t adding hundreds or ones, so only the number in the tens place changes.)

continued on next page
Apply repeated reasoning to the equations in Set D and Set E:

- Let’s use this same reasoning when we talk about the sums in Set E. How can we use what we know about adding 10 to solve for the sum of 123 + 10 = ____? (I added one 10 to the two tens in 123 and got 133.)
- Where are the two tens he is talking about? (The 2 in 123 is in the tens place, so that tells us there are two tens in 123.)
- Who agrees that we can add the one 10 and the two tens together? Say why you agree.
- The sum is correct; 123 + 10 = 133. How can we use this equation to think about 123 + 20 = ____? (There’s one more 10. We had two tens in 20, now there are three tens in 30.)
- The sum of the first equation is 133 and the sum of the second equation is 143. The sum of the second equation is 10 more. If we started out at the same number, why are the two sums different? (We added different amounts of tens, one amount was 10 and the other amount was 20.)
- Okay, let’s look at the next two equations: 123 + 30 = _____ and 123 + 40 = _____. What do you notice about these equations? How can the first two equations help us solve these equations? (Challenging)
- When we are adding numbers, adding the numbers in the same place value together means that this place value changes in the sum. Because we only were adding groups of 10, the tens changed in the sum BUT the numbers in the hundreds place and the ones place does not change as long as we do not reach 90. When we reach 90, one more 10 means we change the tens and the hundreds place. (Recapping)

### Application

Use what we just discussed about using place value to add tens to solve the pairs of equations below.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>68 + 20 = ____</td>
<td>45 + 30 = ____</td>
<td>72 + 10 = ____</td>
</tr>
<tr>
<td>68 + 30 = ____</td>
<td>45 + 40 = ____</td>
<td>72 + 20 = ____</td>
</tr>
<tr>
<td>168 + 20 = ____</td>
<td>145 + 30 = ____</td>
<td>172 + 10 = ____</td>
</tr>
<tr>
<td>168 + 30 = ____</td>
<td>145 + 40 = ____</td>
<td>172 + 20 = ____</td>
</tr>
</tbody>
</table>

### Summary

See RECAPPING.

### Quick Write

Tell how you can use what you know about adding tens to solve 127 + 40 = ____.
Sit-Ups

Candice writes down the sit-ups she does this week. Her mother agreed to give her a sticker if she does at least 50 sit-ups in two days. Does Candice accomplish the challenge?

Candice’s Sit-Ups this Week

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>39</td>
<td>23</td>
<td>27</td>
<td>44</td>
</tr>
</tbody>
</table>

a. How many sit-ups does Candice do on Monday and Tuesday? Show an equation that will let her mother know if she has earned the sticker.

b. How many sit-ups does Candice do on Wednesday and Thursday? Show an equation that will let her mother know if she has earned the sticker.
c. Candice sets a goal of doing over 100 sit-ups next week. Write down one way for Candice to plan to do her sit-ups to reach her goal of over 100 sit-ups.

**Candice’s Sit-Ups**

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If Candice follows this plan, how many sit-ups will she do altogether?
Sit-Ups

Rationale for Lesson: In this lesson, students continue to solve “putting together” situational problems with unknown wholes (sums) but this time, the information is in a table. Prior to this lesson, two strategies were emphasized in the lesson, the “Count On by Tens” strategy and the “Add by Place Value” strategy. In this lesson, students learn another strategy, one related to the concept of conservation called, “Juggle Amounts to Make Friendly Numbers.” (This is the first of two Friendly Number strategies.) While exploring this strategy, students learn that they can move amounts from one addend to another to make quantities that are easier and quicker to add.

Task 4: Sit-Ups
Candice writes down the sit-ups she does this week. Her mother agreed to give her a sticker if she does at least 50 sit-ups in two days. Does Candice accomplish the challenge?

Candice’s Sit-Ups this Week

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>39</td>
<td>23</td>
<td>27</td>
<td>44</td>
</tr>
</tbody>
</table>

a. How many sit-ups does Candice do on Monday and Tuesday? Show an equation that will let her mother know if she has earned the sticker?

See student paper for complete task.

Common Core Content Standards

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

2.NBT.B.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
### Essential Understandings

*Greyed-out portions not addressed in task or lesson.

- Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.
- Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.
- Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.
- A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

### Materials Needed

- Student reproducible task sheet
- Part-part-whole map, hundreds chart, base ten blocks
SET-UP PHASE
Look at the task. Does it look different to you? It should, because this task is giving us information in the form of tables. Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes. Use the counters and your part-part-whole map or create a diagram to show Candice's sit-ups. Remember if you make a diagram that we use a line to show a rod of ten and a square or circle to show one. You can also use a hundreds chart if you would like. Write equations to tell about Candice’s sit-ups.

EXPLORE PHASE

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applies the “Count On by Tens” strategy by skip counting from 39 when solving 39 + 23.</strong></td>
<td>Tell me how you figured out 39 + 23?</td>
<td>Can you repeat your same method when solving for the sum of Wednesday’s and Thursday’s sit-ups?</td>
</tr>
<tr>
<td>Writes 39, 59, then writes 60, 61, 62.</td>
<td>How do you know for sure that you added on all 23?</td>
<td></td>
</tr>
<tr>
<td><strong>Applies the “Add by Place Value” strategy while adding 27 + 44.</strong></td>
<td>What do these numbers tell you about Candice’s sit-ups?</td>
<td>Now plan the number of sit-ups that Candice will do next week and come up with her total. I’ll be back to check in with you about Candice’s sit-ups for next week.</td>
</tr>
<tr>
<td>Writes 27 + 44&lt;br&gt;20 + 40 = 60&lt;br&gt;7 + 4 = 11&lt;br&gt;60 + 11 = 71</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Writes equations.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39 + 23 = 62&lt;br&gt;27 + 44 = 71</td>
<td>Tell me about 62 and 71, what do these numbers mean?</td>
<td>Add information to your work so that anyone who looks at your paper knows what 62 and 71 tell about Candice’s sit-ups.</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Part A, Candice’s sit-ups on Monday and Tuesday

EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

• Someone show us Candice’s sit-ups and tell us what you are trying to figure out. (I showed 39 and 23.)
• Who can add on? I’m not understanding where 39 and 23 came from. (39 are the sit-ups for Monday, 23 are the sit-ups for Tuesday and we have to give the total for two days.)
• We know the two parts we are adding, the two addends, are 39 and 23. We have to find the total—the sum of sit-ups on Monday and Tuesday. (Revoicing)

EU: Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.

Add by Place Value:

• Earlier, we added tens to tens. Can we do that here, add the tens to the tens to find the sum of 39 + 23? What would we do? Before someone explains, can someone show the sit-ups on the overhead with our base ten blocks? (Three tens and two tens is five tens or 50, then 9 + 3 is 12.)
• What do we do once we have 50 and 12?
• Let’s try doing mental math to find this sum. What is 50 + 12? (62)
• We can add the tens to the tens and then the ones to the ones and then we have to put the two sets together. This is the “Add by Place Value” strategy. (Revoicing and Marking)

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.

Count On by Tens:

• Who can use our counting on by tens strategy to find the sum of 39 + 23? (39 and 10 is 49, 49 and 10 more is 59. Then 60, 61, 62.)
• Let’s count out loud and use her counting on by tens strategy. (39 and 10 is 49, 49 and 10 more is 59. Then 60, 61, 62.)
• We can count on by tens from 39. We counted on two more tens and then we had to remember the three ones. (Marking)

Juggle Amounts to Make Friendly Numbers:

• I want to challenge you to use the “Juggle Amounts to Make Friendly Numbers” strategy. I saw someone do this; they juggled amounts to make numbers that were easier to add, they are friendly and easy to work with. Watch what I do with the base ten blocks and see if you can describe what is happening. (Show 39 + 23. Move one from the set of 23 to the set of 39 showing four rods of ten.)
• Who can describe what they see with the tens and ones now? (40 + 22)
• Why do you see 40 and 22 now? (One was moved from 23 making it 22 and now instead of 39, you have 40.)
• Someone else say it again and point to the tens and ones.
• What makes it easier to add 40 + 22 than 39 + 23? (We know that we can do four tens and two tens and get six tens plus the two ones. This is fewer steps than 39 and 23.)
• What makes it okay to add this way, to move an amount from one set to the other set?
• We didn’t take any counters away; all we did was move them from one set to the other, so it is okay to “juggle them” and add because you still have the same amounts. (Recapping)
Part B, Candice’s sit-ups on Wednesday and Thursday, 27 + 44

EU: A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

- What equation can we write to show the sit-ups on Wednesday and Thursday? \((27 + 44 = 71)\)
- Someone wrote \(27 + 44 = 71\). Someone else wrote \(27 + 44 = 71\) sit-ups
  
  Wed. + Thurs. = total sit-ups

  What is the difference between these two answers?

- How does the second example with the label sit-ups and the day written under 27 and 44 help you think about the sit-up situation?

### Add by Place Value:

- Can someone who used the place value strategy explain how they solved the problem?

### Juggle Amounts to Make Friendly Numbers:

- Tell us how you found that answer. \((I\; moved\; 3\; from\; 44\; over\; to\; 27,\; making\; 30\; and\; 41\; which\; is\; 71.)\)
- Who understands what she said and can put her way of solving the problem into your own words and move the tens and ones so we can follow your thinking?
- Why are we allowed to move an amount from one addend to another addend? \((We\; aren’t\; getting\; rid\; of\; any\; or\; getting\; more,\; we\; are\; just\; moving\; 3\; from\; this\; part\; to\; this\; part.)\)

  **(Challenging)**

- I hear you saying that we’re not adding more or subtracting any, we are just moving some from one part to the other part. This means that the total, the sum, stays the same. Is that what you’re saying? **(Recapping)**
- Does that mean we can write \(27 + 44 = 30 + 41\)? What do you think? Turn and talk with your neighbor.
- The expressions are equivalent because each side of the equal sign is 71.

### Application

Solve the equations. Use one of the strategies we talked about today to solve the equations.

- \(39 + 41 = \)____
- \(61 + 29 = \)____
- \(19 + 21 + 31 + 19 = \)____
- \(29 + 19 + 11 + 41 = \)____
- \(111 + 19 = \)____
- \(149 + 31 = \)____
- \(139 + 121 = \)____
- \(121 + 119 = \)____

### Summary

See **RECAPPING** above.

### Quick Write

Look at this expression. Why is the juggling strategy a good strategy to use for this problem?

- \(49 + 24 = \)__

### Support for students who are English learners (EL):

1. Monitor the pace for students who are English learners and check in with them to be sure they are following the discussion. Have students point to the tens and ones on display. Give students opportunities to turn and talk.
2. Ask students who are English learners to physically point to the counters as they talk about the relationship between one addition equation and the next addition equation.
Flowers

Read and solve the story problems below. Make diagrams or use the base ten blocks and write equations to show your thinking.

a. Sandy loves flowers. She grows them in her garden. Below is a list of the summer flowers she grows in her garden. How many summer flowers grow in Sandy’s garden?

<table>
<thead>
<tr>
<th>Sandy’s Summer Flowers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daisies     15</td>
</tr>
<tr>
<td>Tulips      25</td>
</tr>
<tr>
<td>Irises      19</td>
</tr>
<tr>
<td>Roses       22</td>
</tr>
</tbody>
</table>

b. Sandy also grows fall flowers. This is the number of fall flowers that she plans to grow. How many fall flowers grow in Sandy’s garden?

<table>
<thead>
<tr>
<th>Sandy’s Fall Flowers</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Mums   38</td>
</tr>
<tr>
<td>Red Mums     43</td>
</tr>
<tr>
<td>Yellow Mums  25</td>
</tr>
</tbody>
</table>

c. Which type of flowers, the fall flowers or summer flowers, does Sandy grow more of each year?
Flowers

Rationale for Lesson: Like the previous problem, this problem involves work with data in a table and a “putting together” situational problem with an unknown whole (sum). This task differs from previous tasks because it is the first time students are adding three or four sets of data. The set of data is ideal for making use of two of the previously discussed strategies, the “Add by Place Value” strategy and the “Juggle Amounts to Make Friendly Numbers” strategy. In addition to these strategies, a new strategy will be discussed in this lesson, the “Make Ten from Ones” strategy.

Task 5: Flowers
Read and solve the story problems below. Make models and write equations to show your thinking.

a. Sandy loves flowers. She grows them in her garden. Below is a list of the summer flowers she grows in her garden. How many summer flowers grow in Sandy’s garden?

See student paper for complete task.

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>2.OA.A.1</th>
<th>Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.NBT.B.6</td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
</tr>
<tr>
<td></td>
<td>2.NBT.B.7</td>
<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
</tr>
</tbody>
</table>

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
**Essential Understandings**

*Greyed-out portions not addressed in task or lesson.*

- Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.
- The position of the digits 0–9 in a number represents different size amounts within a written number.
- Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.
- Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.
- A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

**Materials Needed**

- Student reproducible task sheet
- Part-part-whole map, hundreds chart, base ten blocks
SET-UP PHASE
Take a look at this task. You can see we are going to be working with information from a table again. Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes. Use the base ten blocks and your part-part-whole mapping device or create a diagram to show Sandy’s flowers. You can use a hundreds chart if you would like. Write equations to tell about Sandy’s flowers.

EXPLORE PHASE

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applies “Add by Place Value” strategy.</td>
<td>Tell me how you came up with 81 flowers.</td>
<td>Does Sandy have more summer or fall flowers?</td>
</tr>
<tr>
<td>Daisies 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tulips 25</td>
<td></td>
<td>How can you figure this out?</td>
</tr>
<tr>
<td>Irises 19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roses 22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Writes 10 + 20 + 10 + 20 = 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 + 5 + 9 + 2 = 21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>81 summer flowers</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Applies “Count On by Tens” strategy, but loses track of the tens and ones and arrives at an incorrect sum.</th>
<th>How did you figure out the total number of summer flowers counting on by tens?</th>
<th>I think I understand how you were able to determine which of them, the summer or fall flowers, was more, but without some evidence of your thinking and how you arrived at the sums for the summer and fall flowers, I don’t know for sure. Add some information to your paper so that it is clear how you arrived at the sum of summer flowers and sum of fall flowers. I will be back.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t get started adding four addends.</td>
<td>Which two sets of flowers can you represent with the tens and ones blocks?</td>
<td>Now that you have added these two sets, what can you do next? Give that a go. I’ll be back to see you how you are doing.</td>
</tr>
<tr>
<td></td>
<td>Add these two sets together. What is the sum?</td>
<td></td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Part A and Part B, adding four two-digit numbers to find sums (All EUs and discussion cycles use the numbers from Part A, all EUs or select EUs can be used to discuss Part B.)

EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

- What did this story problem ask us to find out about Sandy’s flowers? (The total amount of flowers that she grows in the summer and the total number of flowers that she grows in the fall.)
- Yes, we need to find the total number of flowers, the sum of the summer flowers and the sum of the fall flowers. (Marking)
- What did we know about her summer flowers? (We knew the types of flowers she grew and the amount of each type. 15, 25, 19, and 22.)
- You did a great job reading the chart. The chart gives us this information. Sometimes we will have to use information from charts to solve story problems. (Marking)
- What equation did you and your group write to show how to figure out the total number of summer flowers? (15 + 25 + 19 + 22) (Write the equation on the board with a question mark [? in the place of the sum, 15 + 25 + 19 + 22 = ?)

EU: The position of the digits 0–9 in a number represents different size amounts within a written number.

- Can you represent the amounts with the rods and ones?
- Before you show 22, I see two 2s; are they the same? Can someone explain the difference between the first 2 and the second 2? (It may be beneficial to write 22 on the board and touch the first 2 and the second 2 so that everyone is able to follow along.) (The 2 on the right is two ones. The other 2 on the left is two tens.)
- The placement of the number matters. The numbers 0–9 are used over and over but the place, or position, of the numbers matters because each place has a certain value. (Marking) Who can say back what I mean? (The only numbers we use are 0–9. Where the number is used makes a difference.)
EU: Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.

- She chose to use the “Add by Place Value” strategy. But before she shows us her work, everyone try this using the “Add by Place Value” strategy.

  Daisies 15
  Tulips 25
  Irises 19
  Roses 22

  (Have a few people write their work on the board.) Check out how some of your classmates were thinking about the “Add by Place Value” strategy. See if you agree with their method.

  **First Way (Add by Place Value)**  **Second Way (Make a Ten from Ones, introduced in this lesson)**

  \[
  \begin{align*}
  10 + 20 + 10 + 20 &= 60 \\
  5 + 5 + 9 + 2 &= 21 \\
  \end{align*}
  \]

  \[
  \begin{align*}
  10 + 20 + 10 + 20 &= 60 \\
  5 + 5 &= 10 \\
  9 + 1 &= 10 \\
  60 + 10 &= 70 \\
  70 + 10 &= 80 \\
  80 + 1 &= 81 \\
  \end{align*}
  \]

**Add by Place Value:**

- Can someone explain the “Add by Place Value” strategy? We have used this for the last couple of days.
- Does everyone agree and understand what we did? Can someone put it into their own words?
- Notice that there is a step missing. Stop and jot, everyone. What else is needed to finish the problem? *(We need to add 60 and 21 to get 81.)*
- What does the 81 tell us about? *(The summer flowers.)*

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same. *(Conservation of Number)*

**Make a Ten from the Ones:**

- Let’s look at the other group’s work and try to figure out their strategy for solving this problem.

  \[
  \begin{align*}
  10 + 20 + 10 + 20 &= 60 \\
  5 + 5 &= 10 \\
  9 + 1 &= 10 \\
  \end{align*}
  \]

  \[
  \begin{align*}
  60 + 10 &= 70 \\
  70 + 10 &= 80 \\
  80 + 1 &= 81 \\
  \end{align*}
  \]

- What did this group do that is different from the first group? *(This group made groups of 10. Then they added the ones.)*
- Here are the original amounts on the overhead. *(Show with base ten blocks.)* Someone show us what this strategy looks like using base ten blocks.
- They added the numbers that are in the tens place. I heard someone say six, do we mean six groups of 10? *(Revoicing)*
- After they put the tens together, what did they do? *(They took the 5s and put them together to make a 10, and then they put the 1 and the 9 together to make a 10. The two 10s together are 20.)*
- Turn and tell your partner how we came up with 20 ones.
- "Make a Ten from Ones" is a strategy that is very similar to the “Add by Place Value” strategy. The method is similar because the tens are added first and then the ones are added. The difference with these strategies is how we add the ones. When we use the “Make a Ten from Ones” strategy, we make groups of tens from the ones, rather than add all of the ones together at the same time. *(Marking)*
- When we are adding numbers, we can add all the numbers from the same place value together and then put those amounts together, like when we added the tens and then the ones. We can also look in the ones place to see if we can make groups of 10. *(Recapping)*
EU: A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

**Juggle Amounts to Make Friendly Numbers:**
- I saw a group find the sum 81 using this approach. Someone from this group explain to us how you got such friendly numbers. (We moved 5 from 25 to 15, and moved 1 from 21 to 19. Then we had four 20s.)

\[
\begin{align*}
15 + 25 + 19 + 22 &= \quad \text{move 5} \quad \text{move 1} \\
20 + 20 + 20 + 21 &= \\
20 + 20 + 20 + 20 + 1 &= \\
\end{align*}
\]

- Why move the 5 from the 25 to the 15? (To make two twenties.)
- Why move 1 from the 22 to the 19? (We can make another 20.)
- So that was 20, 20, and 20; how did you come up with the fourth 20?
- Who can recap for us how they got the four 20s?
- By moving this 5 to the 15 and this 1 to the 19, you are able to make friendly numbers that are very easy to add. (**Revoicing and Marking**)
- Are we allowed to move amounts to make friendly numbers? Why does this strategy work? (**Challenging**) (We aren’t taking any away. We are just moving things around.)
- We are able to move amounts around between addends. We aren’t adding any more counters or taking any counters away from the whole. This helps us make numbers that are friendly and easier to add. (**Recapping**)

### Application
Solve the problems below. Use at least two of the strategies we talked about today: Add by Place Value, Juggle Amounts to Make Friendly Numbers, or Make a Ten from Ones. Write the name of the strategy you used to solve the problem beside your answer.

\[
\begin{align*}
45 + 25 &= \quad 25 + 35 + 30 &= \quad \\
59 + 11 &= \quad 31 + 49 + 10 &= \\
25 + 19 + 5 + 11 &= \quad 125 + 155 &= \quad \\
41 + 29 + 15 + 35 &= \quad 159 + 111 &= \quad \\
\end{align*}
\]

### Summary
See **RECAPPING** above.

### Quick Write
Study the two part-part-whole maps to the right and write equations for each map. Then using what you know about place value, write about how you know the two maps have the same sum.

![Map 1](image1)

![Map 2](image2)
Task 6

Name_________________________________________________________

**Sports Equipment**

*Marcus counts the basketballs, baseballs, footballs, and soccer balls in the gym. He records the information in the table below.*

<table>
<thead>
<tr>
<th>Sports Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer balls: 28</td>
</tr>
</tbody>
</table>

Marcus claims that he has more basketballs and baseballs than he has footballs and soccer balls.

Do you agree or disagree with Marcus’s claim? Explain why you agree or disagree. Write equations and create diagrams or models to support your explanation.
**Sports Equipment**

*Rationale for Lesson:* This lesson, like the previous lesson, makes use of a “put together” situation and information in a table. The lesson continues to focus on the use of friendly numbers to make addition easier, but a new friendly numbers strategy is introduced. The new strategy is based on the concept of compensation and addresses times when addends are increased to make a friendly number and then that increase must be adjusted for by subtracting the amount of the increase from the resulting sum. The strategy is called, “Making Friendly Numbers and Adjusting.” This is the second friendly number strategy, and it is intentionally introduced so that students realize they have the “right” to think about and apply problem-solving strategies in a way that make sense to them.

**Task 6: Sports Equipment**

Marcus counts the basketballs, baseballs, footballs, and soccer balls in the gym. He records the information in the table below.

<table>
<thead>
<tr>
<th>Sports Equipment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Marcus claims that he has more basketballs and baseballs than he has footballs and soccer balls. Do you agree or disagree with Marcus’s claim? Explain why you agree or disagree. Write equations and create diagrams or models to support your explanation.</td>
<td></td>
</tr>
</tbody>
</table>

*See student paper for complete task.*

**Common Core Content Standards**

- **2.OA.A.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
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**Standards for Mathematical Practice**

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- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
<table>
<thead>
<tr>
<th>Essential Understandings *Greyed-out portions not addressed in task or lesson.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.</td>
</tr>
<tr>
<td>• Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.</td>
</tr>
<tr>
<td>• A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.</td>
</tr>
<tr>
<td>• Altering a number in an addition equation by increasing or decreasing an addend means that the sum will also have to be altered in order to compensate by using the operation opposite from the initial increase or the decrease.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Student reproducible task sheet</td>
</tr>
<tr>
<td>• Part-part-whole map, hundreds chart, base ten blocks</td>
</tr>
</tbody>
</table>
SET-UP PHASE
In this task you are going to think about the claims that Marcus makes. Listen as I read the task. Follow along on your paper. Work by yourself for a few minutes. You have base ten blocks and other tools available to you to help make sense of the situation. You can also make a diagram. I want you to challenge yourself to use one of our strategies listed on the poster. See what you think. Write equations and an explanation to why you agree or disagree with Marcus.

EXPLORE PHASE

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applies “Make a Ten from Ones” strategy to determine the sum of 28 and 22.</td>
<td>Which strategy did you use?</td>
<td>You did a nice job of explaining your work. I’m wondering if you could challenge yourself to think of another strategy you could use to find the sum.</td>
</tr>
<tr>
<td></td>
<td>Tell me how the numbers relate to sports equipment.</td>
<td></td>
</tr>
<tr>
<td>Uses traditional algorithm for adding two-digit numbers, but makes an error.</td>
<td>Tell me about your way of solving for the number of footballs and soccer balls.</td>
<td>You’ve done some great thinking here and I understood what you were trying to do, but the sum you have written on your paper does not match what you explained to me. Is the sum you have written, 410, a reasonable answer for 28 + 22?</td>
</tr>
<tr>
<td>28 + 22</td>
<td></td>
<td>See if you can figure out where you made the mistake. Using base ten blocks may help; I’ll be back to check on your work.</td>
</tr>
<tr>
<td>410</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agrees with Marcus but does not provide an explanation.</td>
<td>Tell me why you agree with Marcus.</td>
<td>These amounts are very close to each other, and lots of people are saying Marcus has the same number of basketballs and baseballs as soccer balls and footballs. Write an explanation or create a diagram that would explain to someone who looks at your paper why you agree with Marcus. We always need to explain our thinking when we are agreeing or disagreeing with someone.</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Understanding Marcus’s claim

**EU:** Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.

- Let’s talk about Marcus's claim. What do we have to do with this problem? (Marcus claims that he has more basketballs and baseballs than footballs and soccer balls.)
- See if you can figure this out. Use your strategies.

Baseballs and basketballs (all EUs or select EUs can be used for footballs and soccer balls)

**EU:** Digits from the same place value position can be combined, the sum of which represents a multiple of the corresponding place value.

*Add by Place Value:*

- What did this group do when figuring out the number of basketballs and baseballs? The group wrote:
  - $20 + 20 = 40$
  - $4 + 7 = 11$
  - $40 + 10 + 1 = 51$
  (They did the tens and got 50, then added the ones and got 51.)

- Who thinks they understand this and can explain it to us and show us with the base ten blocks?
- It sounds like we are saying that $24 + 27 = 20 + 20 + 10 + 1$. (Write equation on the board.) Who can tell us why we can say these two expressions are equal? (Challenging) (They say the same thing. One is spread out and one has the tens and ones together.)
- You added $24 + 27$ and got 51 and then you added $20 + 20 + 10 + 1$ and got 51, which means that both sides of the equal sign are the same amount; they are equal. (Marking and Revoicing)

**EU:** A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

*Juggle Amounts to Make a Friendly Number:*

- How did you figure out how many baseballs and basketballs? (I did 24 and 27 and I made 27 a 30 because it is easier. I got the 3 from the 24 and this is 21 now. So my answer is 51.)
- Who followed what she said and can put her way in your own words? (She crossed out 24 and made it 21 and then took the 3 over to 27 to make this 30.)
- Is this what you did? (Records 27 + 24 as $30 + 21 = 51$.)
- Who can say back how $27 + 24$ became $30 + 21$?
- You can change a number into a friendly number, like when we changed 27 into 30, by taking what you need to do this from the other number you are adding. We can move some from one part to another part to make the numbers easier to add. (Marking)
EU: Altering a number in an addition equation by increasing or decreasing an addend means that the sum will also have to be altered in order to compensate by using the operation opposite from the initial increase or the decrease.

Make Friendly Numbers and Adjust:
- Did anyone use friendly numbers to do this problem? (I made 27 into 30, too, but I didn’t take any from the 24 to do it, I just rounded up.)
- Say more about what you did. (I thought of 27 as 30 and added 30 and 24 to get 54, then I had to take 3 away, and got 51.)
- I heard you say that you took 3 away. Why did you have to take 3 away from 54? (Well, I had to add 3 to 27 to make it 30, so when I was done adding all the numbers, I had to take the 3 away.)
- Hmmmm...let’s see if I have this right. You started with 27 and made it 30 by adding 3 more. Then you added the 30 and 24 to get 54. Then you had to subtract the 3 you added in the beginning, you had to adjust the amount. Is that what you did? (Marking and Revoicing)
- I think that we should take a look at this strategy using base ten blocks. Someone come up and show us what is happening.
- Now who can explain what we just saw? (Hear from at least three students.)
- We know that we can make friendly numbers to make adding easier. We’ve been talking about juggling amounts, moving amounts from one addend to another, for the last couple of days, but today, we see that we can make friendly numbers by adding on to an addend and then subtracting that same amount from the sum. When we add an amount to one of the addends, we have to make sure we adjust the sum in the end. (Marking and Recapping)

Make Friendly Numbers and Adjust, Discussion of an Error:
- When you were working you made an error and I challenged you to explain to the class what might have happened. Mathematicians do this all the time. They are curious about the errors they make because figuring out their errors prevents them from making the same error in the future. Everyone listen and see if you understand what might have happened. (Challenging) (I added 2 at the beginning to make the 28 into 30, and my answer was 52. I forgot to subtract the 2 more that I had added. My answer really should have been 50, not 52. I had 2 more than I needed in my answer.)
- These are our original amounts 28 + 22
  30 + 22 = 52
  50 + 2 = 52 footballs and soccer balls
- Who can say why 52 is not an accurate answer for the total amount of footballs and soccer balls? (30 is two more than 28, which means 2 more were added to the whole; that is too many, so some need to be taken away.)
- Can someone else say this again? (To get 30, they thought of 28 + 2, then 30 + 22, which is 52, but they have to take away the first +2, so it is 52 – 2, so it is 50.)
- Great ideas to think of it as +2 and -2, this really helps us keep track of what we are doing. If we add 2 to something and then take 2 away, have we changed the amount in the whole? (Marking and Challenging)
<table>
<thead>
<tr>
<th>Application</th>
<th>Solve the equations. Make use of the strategies we have been exploring to help you solve the equations.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>48 + 22 = ___ 12 + 24 + 34 = ___ 132 + 158 = ___ 29 + 41 = ___ 55 + 22 + 13 = ___ 146 + 124 = ___</td>
</tr>
<tr>
<td>Summary</td>
<td>When you add in more than the problem shows, add to the addends, then you have to adjust the final amount by subtracting the amount that was added in the beginning. See RECAPPING above.</td>
</tr>
<tr>
<td>Quick Write</td>
<td>Look at the equations. Name a friendly number that you would make in each of the equations and explain how you would make it.</td>
</tr>
<tr>
<td></td>
<td>68 + 35 = ___ 27 + 57 = ___ 38 + 19 = ___</td>
</tr>
</tbody>
</table>
Fieldtrip to the Zoo

The principal at an elementary school is organizing a fieldtrip to the zoo for first, second, and third grade students.

Help the principal figure out the total number of students going to the zoo for each grade level. Fill in the unknown totals in the table.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Classroom A</th>
<th>Classroom B</th>
<th>Total Number of Students Going to the Zoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>48</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td>53</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td>24</td>
<td>47</td>
<td></td>
</tr>
</tbody>
</table>
Fieldtrip to the Zoo

Rationale for Lesson: Like previous lessons, this lesson involves a “put together” task and work with a table. At this state, students are asked to read and interpret the information in the table. In this task, students are given multiple pieces of information in a table. They are dealing with three grade levels and two classrooms in each grade level. The numbers have been selected to encourage the use of strategies that make friendly numbers. There are two friendly number strategies that students can make use of in this task: “Juggling Amounts to Make Friendly Numbers” (related to the concept of conservation) and “Make Friendly Numbers and Adjust” (related to the concept of compensation). Several strategies should be discussed during the discussion phase of the lesson and then the lesson should end by challenging students to use a strategy that they have not yet tried to solve for the number of third grade students attending the zoo. Honoring students’ strategies by giving them the opportunity to display them and compare and contrast the strategies should be the goal of the lesson.

Task 7: Fieldtrip to the Zoo
The principal at an elementary school is organizing a fieldtrip to the zoo for first, second, and third grade students. Help the principal figure out the total number of students going to the zoo for each grade level. Fill in the unknown total in the table.

See student paper for complete task.

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>2.OA.A.1</th>
<th>Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</th>
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<td>2.NBT.B.6</td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
<td></td>
</tr>
<tr>
<td>2.NBT.B.7</td>
<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
<td></td>
</tr>
</tbody>
</table>

Standards for Mathematical Practice:
- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
<table>
<thead>
<tr>
<th>Essential Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.</td>
</tr>
<tr>
<td>• Digits from the same place or position can be combined, the sum of which represents a multiple of the corresponding place value.</td>
</tr>
<tr>
<td>• Two or more expressions/equations are equivalent if they represent the same quantity.</td>
</tr>
<tr>
<td>• Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same.</td>
</tr>
<tr>
<td>• A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.</td>
</tr>
<tr>
<td>• Altering a number in an addition equation by increasing or decreasing an addend means that the sum will also have to be altered in order to compensate by using the operation opposite from the initial increase or the decrease.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Student reproducible task sheet</td>
</tr>
<tr>
<td>• Part-part-whole map, hundreds chart, base ten blocks</td>
</tr>
</tbody>
</table>
SET-UP PHASE
This task is a little different than the ones we’ve been doing. Take a look at the table. This table shows the number of students going on a fieldtrip to a zoo from grades one, two, and three. There are two first grade classrooms, Classroom A and Classroom B. Put your finger on Classroom A and Classroom B. Help the principal complete the table by finding the number of students going from first grade, the number of students going from second grade, and the number of students going from third grade. Work by yourself for a few minutes. Use the base ten blocks and your map or create a diagram to show your work.

EXPLORE PHASE

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applies “Count On by Tens” strategy, uses hundreds chart.</td>
<td>How are you using the hundreds chart to figure out the students going on the fieldtrip from first grade?</td>
<td>Write an equation that describes how you started at 48 on the hundreds chart and then kept adding tens.</td>
</tr>
<tr>
<td>Complates the table (shows no diagrams or equations).</td>
<td>How did you find that there are 83 first grade students going on the fieldtrip to the zoo?</td>
<td>All of the totals are correct. You have accurate calculations. Since you did such a great job, I’m going to challenge you to write an explanation for how you used mental math to figure out 48 + 35. Can you make 48 into a friendly number and then solve the problem?</td>
</tr>
<tr>
<td>Applies “Make Friendly Numbers and Adjust” strategy.</td>
<td>How do the equations describe the students who are going on the fieldtrip to the zoo?</td>
<td>Complete the table (if necessary) and write an explanation so that someone who reads your paper will understand what you know about the fieldtrip and who is going.</td>
</tr>
<tr>
<td>Determines the number of students in each grade who are going on the fieldtrip.</td>
<td>How did you find the number of students from each grade level going on the fieldtrip?</td>
<td>You really took your time and thought through the task. I’m going to give you a challenge. Are you ready? See if you can figure out how many students in first, second, and third grade are going on the fieldtrip.</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

Understanding the information in the table:
- Let’s talk about this table. What does this table show? *(The table shows the number of students going on the trip from each classroom.)*
- Who can say more? What else does the table show you?
- You are correct. These two columns show us the number of students going on the fieldtrip from each classroom. First, second, and third grade each have two classrooms, Classroom A and Classroom B. *(Revoicing)*
- I’m wondering about this column. It is blank; what goes in this column? *(The total number of students for each grade.)*
- We just read the table. Did you know that? Why is it important for us to be able to “read” the table? *(Challenging) We need to know what we are doing so we have to read the table.)*
- What does it mean to read the table? What in the table told you about the story problem? *(The top and the sides.)*
- You’re right, the bold headings that say the grade and the classrooms. *(Marking) How is the title helpful? It tells you what the whole thing is about.)*

First grade, 46 + 35 *(EUs and similar discussion cycle can be repeated for second and third grade)*

**EU: Part-part-whole relationships exist within both addition and subtraction and because of these relationships, when one quantity is unknown, known quantities can then be used to determine the unknown.**
- What do we know about the first graders? What are we trying to find out? *(We know 48 and 35. We need to find the number of students going on the fieldtrip from first grade.)*
- You gave us the right numbers; we know there are 48 students from Classroom A and 35 students from Classroom B going to the zoo. We need to find the sum of these two classrooms. *(Revoicing)*
- How many first graders are going on the trip? *(83) What equation did you write? (48 + 35 = 83)*

**EU: Digits from the same place or position can be combined, the sum of which represents a multiple of the corresponding place value.**

*Add by Place Value:*
- Show us how you found the sum of 83. *(I added the four tens and the three tens together and then I added the 8 and 5 ones together.)*
- At this stage of our work, I want the class to describe the strategy completely without my help. Can others add on?
- When we apply the “Add by Place Value” strategy, we add all of the numbers from the same place value together. We add the tens and tens, the ones and ones. Then we add the tens and the ones together. *(Marking)*
- Let’s try applying the “Add by Place Value” strategy to figure out the second grade students going to the zoo. Stop and jot. Everyone show work for the place value strategy. Write down what you do so you can describe it to others.
- What did you do? *(50 + 30 = 80 and 3 + 9 = 12 and 80 + 12) (Show the amounts and call on at least five students to explain the place value strategy.)*
- What do we think about applying the “Add by Place Value” strategy to solve 39 + 53? Is there another strategy, one that some people think would make our work easier? If you make this claim, be prepared to support your claim. *(Challenging)
Second grade, 53 + 39 (EUs and similar discussion cycle can be used for first and third grade)

EU: A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

EU: Two or more expressions/equations are equivalent if they represent the same quantity.

Juggle the Amounts to Make Friendly Numbers and Count On by Tens:
- How would you solve 39 + 53 with the “Juggle the Amounts to Make Friendly Numbers” strategy? Stop and jot. Do the work and be prepared to talk about your method.
- Come up and show us with the base ten blocks what you did when you describe your way of doing the problem. (After I moved 1 from 53 to 39, I had 52 and 40, so I started at 52 and counted on by four tens.) Count that for us. (52, 62, 72, 82, 92.)
- Who understands what she did and can put the ideas into their own words?
- You used two strategies. You used the “Juggle the Amounts to Make Friendly Numbers” strategy when you moved 1 from 53 to 39 so that you had 52 and 40. (Cross out 53 and write 52 below it, cross out 39 and write 40 below it.) Then you used “Counting On by Tens” from 52. Do I have this right, is this what you did? (Revoicing)

Equivalency and Juggle the Amounts to Make Friendly Numbers:
- To solve the problem 39 + 53, someone wrote this 59 + 33. What did this person do? (This person switched the 50 and the 30.) Is this what you did? (Yes.) Are we permitted to do this? Why?
- Use the “Juggle the Amounts to Make Friendly Numbers” strategy to add these numbers, 59 + 33. Turn and talk.
- Tell us what you did. (I made the 59 into 60 and I did this by taking 1 from the 33 so now it is 32. Then I added 60 + 32 and got 92.)
- What do others think? Do you agree or disagree?
- Who can put the strategy into your own words?
- Was it easier to add 39 + 53 or 59 + 33 and what is your reasoning? (59 + 33 because it made 60 + 32 and even if you have to count 30 more, you can do this easier than counting 50 more.)
- What if I round 59 to 60 and then added 33 and got 93? What would I have to do? (You would subtract 1 because you added 1 to 59 to make it 60, and you didn’t take the 1 from 33 like you did with the last problem.)
- If we add on more, then in the end we must compensate or make adjustments and subtract the amount we added. This is one of our strategies; which strategy is it? (Making Friendly Numbers and Adjusting.) (Marking)
Third grade, 24 + 37 (EUs and similar discussion cycle can be used for first and second grade)

EU: Quantities can be combined, decomposed, and rearranged in any order, but the total number of objects remains the same. (Conservation of Number)
EU: Digits from the same place or position can be combined, the sum of which represents a multiple of the corresponding place value.
EU: Altering a number in an addition equation by increasing or decreasing an addend means that the sum will also have to be altered in order to compensate by using the operation opposite from the initial increase or the decrease.

Relating problem-solving strategies:

- Show two ways of solving this problem. Be prepared to explain why your strategy is the best one to use for the size of the numbers.
- I had three people write their strategies on the board. See if you can repeat their way and name the strategy. I will ask you how the strategies differ from each other. (Have the following strategies displayed. If students do not use three different strategies, then you may show one of the strategies and claim it as your strategy.)

  - 24 + 37
  - 20 + 30 = 50
  - 24 + 40 = 64
  - 21 + 40 = 61
  - 7 + 4 = 11
  - 64 – 3 = 61
  - 50 + 11 = 61

- Can each group explain how they solved the problem and use the blocks to show what they did? I will ask others to say back the strategy and to name it.
- How are the strategies related to each other? (We got the same sum in each one.)
- How do the strategies differ from each other? (One way of solving the problem is the juggling amounts to make friendly numbers, there is the adjusting strategy, and another strategy is adding by place value.)
- What do you think about using all of these ways? Is this a good thing to do? Why or why not?
- Which ways do you understand? Which ways are you still working on and would like to work with a partner who knows the strategy?

Application

Use the listed strategy to write and solve equations.

Make Friendly Number Strategies (Juggle or Adjust)

- 69 + 32 = _____ + _____ = _____
- 46 + 55 = _____ + _____ = _____

Making a Ten from Ones

- 23 + 27 = _____ + _____ + _____ = _____
- 36 + 15 = _____ + _____ + _____ = _____

Add by Place Value

- 66 + 32 = _____ + ______ = _____
- 43 + 55 = _____ + ______ = _____

Summary

There are many ways to solve addition problems; some make use of counting while others make use of friendly numbers or place value. The strategy you choose to use should depend on the numbers you are adding and how comfortable you are with the strategy.

Quick Write

Choose a strategy and explain to the principal how to use the strategy to find the total number of students going on the fieldtrip to the zoo. (83 + 92 + 61)
## Flexible Thinking

*Study each set of expressions below. Explain what changes with each expression in the set. Use diagrams and words in your explanation. Write the sum of the expressions.*

<table>
<thead>
<tr>
<th>Problem A</th>
<th>Problem B</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 + 54 = ___</td>
<td>29 + 52 = ___</td>
</tr>
<tr>
<td>80 + 9 = ___</td>
<td>59 + 22 = ___</td>
</tr>
<tr>
<td></td>
<td>60 + 21 = ___</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem C</th>
<th>Problem D</th>
</tr>
</thead>
<tbody>
<tr>
<td>47 + 33 = ___</td>
<td>19 + 56 = ___</td>
</tr>
<tr>
<td>50 + 30 = ___</td>
<td>20 + 55 = ___</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem E</th>
<th>Problem F</th>
</tr>
</thead>
<tbody>
<tr>
<td>54 + 28 = ___</td>
<td>72 + 28 = ___</td>
</tr>
<tr>
<td>54 + 30 = ___</td>
<td>70 + 30 = ___</td>
</tr>
<tr>
<td>___ − 2 = ___</td>
<td></td>
</tr>
</tbody>
</table>
## Try It:

Find the sum for each equation using one of our strategies. Then explain how you found the sum in words or with new equations or by making diagrams.

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$72 + 18$</td>
<td>______</td>
</tr>
<tr>
<td>2</td>
<td>$39 + 46$</td>
<td>______</td>
</tr>
<tr>
<td>3</td>
<td>$157 + 26$</td>
<td>______</td>
</tr>
<tr>
<td>4</td>
<td>$145 + 126$</td>
<td>______</td>
</tr>
</tbody>
</table>
## Flexible Thinking

**Rationale for Lesson:** Students solidify their understanding of the different problem-solving strategies used in the set of related lessons by first analyzing and naming the strategies, and then applying the different strategies to a set of problems.

### Task 8: Flexible Thinking
Study each set of expressions below. Explain what changes with each expression in the set. Use diagrams and words in your explanation. Write the sum of the expressions.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Expression</th>
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<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>35 + 54 = ____</td>
<td>29 + 52 = ____</td>
<td>19 + 56 = ____</td>
<td>72 + 28 = ____</td>
</tr>
<tr>
<td></td>
<td>80 + 9 = ____</td>
<td>59 + 22 = ____</td>
<td>20 + 55 = ____</td>
<td>70 + 30 = ____</td>
</tr>
<tr>
<td>C</td>
<td>47 + 33 = ____</td>
<td>50 + 30 = ____</td>
<td>50 + 30 = ____</td>
<td>50 + 30 = ____</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>54 + 30 = ____</td>
<td>54 + 30 = ____</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>___ – 2 = ____</td>
<td>___ – 2 = ____</td>
</tr>
</tbody>
</table>

See student paper for complete task.

### Common Core Content Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.NBT.B.6</td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
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<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
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</tbody>
</table>

*Greyed-out portions not addressed in task or lesson.

### Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Practice</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP1</td>
<td>Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>MP3</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>MP4</td>
<td>Model with mathematics.</td>
</tr>
<tr>
<td>MP6</td>
<td>Attend to precision.</td>
</tr>
<tr>
<td>MP7</td>
<td>Look for and make use of structure.</td>
</tr>
<tr>
<td>MP8</td>
<td>Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
### Essential Understandings

- Digits from the same place or position can be combined, the sum of which represents a multiple of the corresponding place value.
- A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

### Materials Needed

- Student reproducible task sheet
- Part-part-whole map, hundreds chart, base ten blocks
SET-UP PHASE
This task is a little different from others that we have done. Today, we are going to be thinking about the strategies we have used to find sums and why we are able to use them. We will do the first part together and then you will do the second part of the task independently first and then discuss your strategies in small groups.

EXPLORE PHASE FOR THE SECOND PART (INDEPENDENT PART) OF THE TASK

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applies “Juggle the Amounts to Make Friendly Numbers” strategy by creating a diagram of tens and ones but fails to cross out and indicate when amounts are moved.</td>
<td>How does your diagram show the strategy you used to solve the problem?</td>
<td>Your diagram just shows the amounts you added, but we don’t know what amounts you moved. Your work should show your thinking, so make arrows, cross out numbers, or write about how you used the strategy to solve the problem.</td>
</tr>
<tr>
<td>Struggles to use a strategy with the numbers in the hundreds.</td>
<td>Tell me how you are thinking about adding the numbers in the hundreds.</td>
<td>It is hard to think about adding 145 + 126. How would thinking about adding 45 + 26 first help you solve the problem? Then you can always add in the 100 + 100. I’ll be back to see how it’s going.</td>
</tr>
<tr>
<td>Has a correct strategy for each problem.</td>
<td>Tell me about the strategy that you used to solve this problem with the hundreds. 145 + 126</td>
<td>Suppose I give you this problem. Can you use our strategies to find the sum? 1259 + 1121?</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE FOR THE FIRST PART OF THE TASK

Discussion of Problem A:
- What strategy is being used for Problem A?
  35 + 54 = ___
  80 +  9 = ___
  *(Add by Place Value strategy)*
- How do you know it is the “Add by Place Value” strategy?

Discussion of Problem B:
- What strategy is being used for Problem B and how do you know?
  29 + 52 = ___
  59 + 22 = ___
  60 + 21 = ___
  *(Switched the tens and then juggled to make 60 by taking 1 from the 21.)*
- Who agrees? Who disagrees? Why do you agree or disagree?

Discussion of Problem C:
- What strategy is being used for Problem C and how do you know?
  47 + 33 = ___
  50 + 30 = ___
  *(Juggle the Amounts to Make Friendly Numbers and then adding)*
- Who agrees? Who disagrees? Why do you agree or disagree?

Continue Discussions for Problem D, E, and F

SHARE, DISCUSS, AND ANALYZE PHASE OF THE SECOND SET OF PROBLEMS

Discussion of two different strategies for 72 + 18 = _____

EU: Digits from the same place or position can be combined, the sum of which represents a multiple of the corresponding place value.

*Add by Place Value:*
- Tell us what you did and show us with the tens and ones.
  72 + 18 = 90
  70 + 10 = 80
  2 + 8 = 10
  80 + 10 = 90
- What strategy did she use when solving the problem? *(The “Add by Place Value” strategy.)*
- Someone say how you know it is the “Add by Place Value” strategy. *(We did the tens and then the ones and then put them together.)*
EU: A quantity in a set can be moved to the other set and the sets can be combined, but the whole amount will remain the same because no additional items were added or taken away.

Juggle the Amounts to Make Friendly Numbers:
- Someone else had a different way of adding 72 + 18. Share with us how you solved the problem. (I knew that 8 + 2 was 10 so I moved 8 over to 72 to make 80 and this left 10, so then I added 80 + 10.)
  \[ 72 + 18 = 90 \]
  \[ 80 + 10 = 90 \]
- Who understands and can put her strategy into their own words?
- You juggled the amounts to make numbers that were easier to add. (Marking)

Repeat similar discussion for two different strategies used for the other equations.
39 + 46 = 
157 + 26 = 
145 + 126 = 

Application
No application.

Summary
When we are adding numbers, we can move amounts from one set to the other set and it will not change the whole.

When we add two-digit numbers, we can use place value to find the sums by adding all of the tens together and all of the ones together, and then add those amounts to find the sum.

When we are solving addition problems, we should look at the numbers to decide which strategy makes the most sense to use.

Quick Write
Study the two part-part-whole maps below. Write an equation for each map and explain why the equation from Map 1 is related to the equation on Map 2.

Support for students who are English learners (EL):
1. Refer back to the running list of solution paths for adding two-digit numbers and ask students who are English learners to identify those that they know how to use and those that they do not know how to use and then let them partner with someone who is proficient with using the strategy that they have set as a goal.
2. Let students partner with other students from whom they want to learn a particular strategy during the Explore Phase. Then hold the student accountable for talking about the strategy.